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PRECONDITIONING FOR FIRST-ORDER SPECTRAL DISCRETIZATIONS

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Introduction

Efficient solution of the equations from spectral discretizations is essential if the high-order accuracy of these methods is to be realized in practice. Direct solution of these equations is rarely feasible, thus iterative techniques are required. As a consequence of the ability of spectral methods to capture accurately a wide bandwidth of information, the eigenvalue spread of a spectral operator is large, typically on the order of the square of a corresponding low-order finite-difference operator. Therefore, an explicit method would converge quite slowly, and the convergence rate would deteriorate very rapidly with mesh refinement. Implicit methods are not efficient, since spectral operators are full, rather than banded as in the case of finite difference.

An alternative to pure explicit iterative schemes is a preconditioned scheme, where explicit iteration of some form is driven not by the spectral residual, but by the residual obtained after some processing is applied to reduce its eigenvalue spread. As will be shown later, this process can also be thought of as an approximate-implicit scheme, which can give some insight into a relevant preconditioning operator.

Preconditioned-iteration schemes for spectral collocation discretization of second-order equations are well-known and proven. For instance, the time-accurate incompressible Navier-Stokes simulations of certain fluid mechanical phenomena, in which Chebyshev collocation is used in two coordinate directions, require the solution of a number of Helmholtz or Poisson equations per time step (ref. 1). The preconditioned scheme used in that work requires less than one second to accomplish such a solution on a 65 x 65 mesh on a CYBER 205 vector computer. The preconditioning operator for these second-order equations is the low-order central finite difference operator, using the Chebyshev collocation points as its mesh. Orszag (ref. 2), originally suggested such preconditioning and provided some analysis for the case of Fourier discretization.

The steady-state compressible Euler or Navier-Stokes equations at high Reynolds numbers, however, are advection-dominated. A spectral solution technique for such equations must therefore deal with operators which are predominantly first-order. Since inversion of the finite-difference preconditioning operator is (as will be shown) related to the finite-difference solution to the original problem, one would like to draw on finite-difference experience in solving compressible Navier-Stokes problems to yield efficient solution of the preconditioning operator. Variations of the Beam and Warming scheme (refs. 3, 4) are popular for such solutions. However, the advection terms in this scheme are central differenced. Elementary analysis of preconditioning first-order Fourier discretization with central finite-difference on the collocation mesh indicate that the convergence rate of such a scheme would be unacceptable; the eigenvalue spread of this preconditioned operator is unbounded. A similar situation appears in the case of Chebyshev discretization, as will be shown here.

A preconditioning scheme for first-order Chebyshev collocation operators is proposed herein, in which the central finite-difference mesh is finer than the collocation mesh. (The authors later found that Orszag conjectured such a preconditioner in an early paper for Fourier series, although no rigorous analysis was given (ref. 2).) Details of the proper techniques for transferring information between the meshes is given here, and the scheme is analyzed by examination of the eigenvalue spectra of the preconditioned operators, corresponding to a pure first-order and to an advection-dominated advection/diffusion problem, both with realistic boundary conditions. The effect of artificial viscosity required in the inversion of the finite-difference operator is examined. A second preconditioning scheme, involving a high-order upwind finite-difference operator of the van Leer type (ref. 5), is also analyzed to provide a comparison with the present scheme. Finally, the performance of the present scheme is verified by application to several test problems.

Overview of Preconditioning Iteration Schemes

Consider the following linear equation

$$L_{sp}\, u = f \tag{1}$$

where the operator L_{sp} is derived from spectral collocation of a differential equation. An iterative scheme is to be used to solve this equation. Given a current estimate of the solution u^n at iterate "n", a simple Richardson iteration scheme for computing a better estimate u^{n+1} is

$$u^{n+1} = u^n - \omega\, (L_{sp}\, u^n - f) \tag{2}$$

where the scalar relaxation factor ω may be chosen either experimentally or via a requirement that some norm of the residual

$$R^n = L_{sp}\, u^n - f \tag{3}$$

be minimized at each step. Rewriting the scheme (2) as

$$u^{n+1} - u^n = \Delta u^n = -\omega\, R^n \tag{4}$$

shows the explicit nature of this iteration. Preconditioning involves choosing an operator M which is more easily invertible than L_{sp}, and for which the scheme

$$\Delta\, u^n = -\omega\, M^{-1}\, R^n \tag{5}$$

converges more rapidly than the scheme (4). The convergence rate of such schemes is quantified in the following way: expand Eq. (5) as

$$u^{n+1} = (I - \omega\, M^{-1}\, L_{sp})\, u^n - \omega\, M^{-1}\, f \tag{6}$$

where I is the identity operator. Subtracting the discrete solution to Eq. (1) (the desired solution u) from both sides of (6), and adding to the right hand side

$$\omega\, M^{-1}\, (L_{sp}\, u - f)\ , \tag{7}$$

which is equal to zero by (1), yields an equation for the discrete error:

$$(u^{n+1} - u) = (I - \omega\, M^{-1}\, L_{sp})\, (u^n - u) \tag{8}$$

For the preconditioner scheme (5) to be convergent, all norms of $(u^n - u)$ must decrease; thus

$$\| I - \omega\, M^{-1}\, L_{sp} \| < 1 \tag{9}$$

is required. Given a preconditioning operator M, the relaxation factor ω is used to satisfy Eq. (9).

From Eq. (9) may be seen what constitutes an effective preconditioner: if the eigenvalues of $M^{-1} L_{sp}$ are clustered in a unit circle centered at 1 in the complex plane, then all error components will converge at nearly the same rate, and an optimal ω may be chosen which will yield rapid convergence. A slow scheme is characterized by a wide spread of eigenvalues from such a clustered pattern.

As an aside, a preconditioned iteration scheme may be looked at as an approximate implicit scheme in the following way. Ideally, given an estimate u^n, one wants the residual at the next iterate to be zero; expanding

$$R^{n+1} = L_{sp} (u^n + \Delta u^n) - f = 0 \tag{10}$$

or

$$L_{sp} \Delta u = -R^n \tag{11}$$

However, L_{sp} is difficult to invert; approximate it on the left by a more easily-inverted operator $\omega^{-1} M$, giving

$$\Delta u^n = -\omega M^{-1} R^n \tag{12}$$

which is identical to Eq. (5). Thus the better $\omega^{-1} M$ approximates L_{sp}, the faster the scheme will converge.

The inverse of the preconditioning operator may be obtained and used directly, as implied by Eq. (5) or the preconditioning equation

$$M \Delta u^n = -\omega R^n \tag{13}$$

may be inverted iteratively. This procedure is beneficial when the spectral operator L_{sp} is nonlinear, necessitating a different operator M at each iterate, or when M is still expensive to invert. An example of the latter case is when Eq. (1) results from spectral discretization of a PDE in two or more dimensions. The corresponding finite-difference discretization operator

works well as a preconditioner (with the modifications described in the next section for first-order operators), but may still be expensive to invert, due to its size. An iterative technique for computing the Δu^n is more efficient than direct inversion for this case, using, for instance, an approximate-factorization scheme. Experience indicates that the preconditioner-inversion iteration need not be driven to convergence for the overall preconditioned scheme (Eq. (5)) to converge rapidly.

Preconditioners for First-Order Spectral Operators

As mentioned in the introduction, the preconditioning scheme investigated here uses a central finite-difference operator, constructed on a mesh finer than that of the spectral discretization. A simple example will show why preconditioning using central finite-differences on the collocation mesh is inadequate. For the model scalar problem $U_x = f$ with periodic boundary conditions on $[0, 2\pi)$, the eigenvalues of the Fourier collocation operator $\partial/\partial x$ are $i\alpha\Delta x$, where α is the wavenumber and Δx is the constant collocation mesh spacing. The product $\alpha\Delta x$ falls in the range $0 < |\alpha\Delta x| < \pi$. The corresponding eigenvalues for the central-difference operator are $i \cdot \sin(\alpha\Delta x)$. Note that as $\alpha\Delta x \to \pi$ the ratio of these eigenvalues is unbounded. This ratio corresponds to the eigenvalues of the preconditioned operator, denoted $M^{-1} L_{sp}$ in the previous section. Such a preconditioned scheme is thus unconditionally unstable, with unbounded growth of the highest wavenumber error components. The use of a finite preconditioning grid averts this unbounded component by introducing some natural dissipation in the preconditioner at the highest wave number of the spectral operator.

Because of the difference between the spectral and preconditioner meshes,

a scheme is needed for transferring information between them. In multigrid terminology, a prolongation operator is needed to transfer the spectral residual which acts as the source term of the preconditioning equation (Eq. (13)), and a restriction operator is needed to compute the solution updates on the spectral mesh. Naturally, it is desirable to transfer as much information as possible form the spectral residual to the preconditioning equation; spectral interpolation is therefore used for the prolongation operator. In the restriction operation, however, aliasing of correction components, with wavenumber higher than that of the spectral mesh must be avoided. Therefore, spectral restriction cannot be used; low-order Lagrange interpolation is used here.

The preconditioning scheme proposed here proceeds as follows for the spectral discretization of a simple model problem $U_x = f$. At each iterate, compute the residual (Eq. (3)) defined on N_{sp} points in the domain. This information is transferred to the (finer) preconditioning mesh via the spectral interpolation operator I_{sp}^{FD}. Denoting

$$\widetilde{R} = I_{sp}^{FD} R \tag{14}$$

the preconditioning equation becomes

$$\widetilde{M}\, \Delta\widetilde{u} = \widetilde{R} \tag{15}$$

where $\Delta\widetilde{u}$ is the update on the preconditioning mesh. For the model problem $U_x = f$ in $x \in [-1,1]$ considered here, the preconditioner $\widetilde{M}$ must approximate the first derivative operator via central finite differences. We use

$$\widetilde{M} = \delta_x^\circ - \varepsilon\, \delta_{xx}^\circ \tag{16}$$

using standard divided-difference notation. The second-difference term is required to avert the odd-even uncoupling of the pure central first-difference operator, and to aid in its inversion. Some type of artificial viscosity is

essentially always required when inverting this type of operator, including application in the Beam and Warming scheme. Note that this artificial viscosity does not effect the spectral solution, being confined to the preconditioner. The magnitude of the parameter is quite small, generally 10^{-3} or less; its effect will be discussed in the next section.

Since we are concerned here with general, nonperiodic problems, the effect of boundary conditions on the preconditioning scheme is relevant. The appropriate spectral boundary condition for this problem is Dirichlet on one end of the domain ($x = -1$). The other end ($x = 1$) requires no boundary condition; the spectral scheme is used to enforce the equation at that point. On the other hand, extrapolation conditions are commonly used in finite-difference discretizations of such problems. Such a boundary condition is used in the preconditioner as analyzed in the next section.

The final step of this preconditioning scheme is to carry the update information to the spectral mesh. The iterate thus concludes with

$$\Delta u = \tilde{I}_{FD}^{sp} \, \Delta\tilde{u}$$

and

$$u^{n+1} = u^n + \omega \, \Delta u \tag{17}$$

where the operator $\tilde{I}_{FD}^{sp}$ uses low-order Lagrange interpolation. For analysis purposes, the above sequence may be collected into a single operator:

$$M^{-1} = \tilde{I}_{FD}^{sp} \, \tilde{M}^{-1} \, I_{sp}^{FD} \tag{18}$$

This is, in effect, the preconditioning operator applied in the above scheme.

In practice on a real problem of interest, the finite-difference operator M is also too large to invert directly at reasonable cost. Preconditioning operators are therefore inverted iteratively in all but the simplest one-

dimensional test problems. Experience with both first- and second-order operators indicates that full machine-zero convergence of this iterative inversion is not necessary; depending on the nature of the iterative scheme, only two orders of magnitude reduction in the residual is necessary.

One preconditioning operator which is attractive from the standpoint of there being a rapid and efficient inversion technique involves high-order upwind differencing of the van Leer type (ref. 5). In this scheme, the flux is split according to direction of propagation, and appropriate upwind differences are taken. The inherent artificial viscosity in such schemes has been shown to provide enhanced robustness, and allows for nearly fully-implicit treatment of the artificial viscosity during iteration for even complex problems, which speeds convergence.

A simple first derivative, taken as a forward-propagating flux, is approximated as

$$\delta^{-}u^{+} = [u^{+}_{i+1/2} - u^{+}_{i-1/2}] \; / \; \Delta x \tag{19}$$

where

$$u^{+}_{i\pm1/2} = u_i \pm \phi/4 \; [(1 \mp k) \; \nabla + (1 \pm k) \; \Delta] \; u \tag{20}$$

The operators ∇ and Δ are undivided backward and forward differences, respectively. The parameter k determines the accuracy of the operator: k=-1 corresponds to a fully-upwind scheme, k=1 to a central difference operator, k=1/3 to a third-order upwind biased scheme.

Results

To investigate the effectiveness of the proposed preconditioning scheme, the eigenvalues of the operator $M^{-1}L_{sp}$ were computed for a number of combinations of N_{sp} and N_{FD}, for the model problem

$$U_x = \sin \pi x, \quad x \in [-1,1] \tag{21}$$
$$U(-1) = 0$$

These results were compared qualitatively with convergence rate observations from actual implementation of the scheme.

Some basic characteristics of the eigenvalue spectra for this problem are as follows: The eigenvalues of the spectral operator L_{sp} alone are dominantly imaginary, with the magnitude of the largest growing as N_{sp}^2 When the spectral operator is preconditioned with the central finite-difference operator on the same mesh, the eigenvalues of the overall operator become dominantly real, which is a desirable feature for use in an iterative scheme. The real parts of these eigenvalues are all positive, the smallest being near 1 and the largest on the order of 50 for the grids investigated. The eigenvector associated with this largest eigenvalue is highly oscillatory, as expected from the discussion in the previous section. As N_{FD} is increased relative to N_{sp}, this eigenvalue pattern generally collapses onto the point (1,0) in the complex plane, with both real and imaginary parts decreasing. This collapse is at first rapid as N_{FD} is increased from N_{sp}, then slows as the eigenvalue with the largest real part moves to the interior of the unit circle centered at 1. There is a single exception to this clustered pattern, that of a small (O(.1)) eigenvalue which remain essentially

fixed for all $N_{FD} < N_{sp}$. Its eigenvector indicates that this eigenvalue is associated with the different conditions used at x = 1 by M and L_{sp}. In general, though, the eigenvalues of the preconditioned operator are strongly clustered, thus one would expect rapid convergence of the iterative scheme.

In Table I are shown the maximum and minimum eigenvalues for the preconditioned operator from the test problem described above, for various combinations of N_{sp} and N_{FD}. The range of N_{sp} considered covers what we expect to be required in spectral discretizations of practical aerodynamic problems. As can be seen, the maximum eigenvalue is large when $N_{FD} = N_{sp}$, and drops rapidly as N_{FD} is increased. The operator appears well-conditioned, that is, the eigenvalues are tightly grouped inside the unit circle centered at 1, when $N_{FD} \geq 1.5\ N_{sp}$ for all of the grids considered. These results were produced with first-order Lagrange interpolation in the restriction operator, and with the artificial-viscosity coefficient used in M fixed at 10^{-3}. Virtually identical results were obtained for $\varepsilon = 10^{-4}$, and for second-order restriction.

In Table II are shown the convergence rates observed from application of this preconditioning scheme to the model problem Eq. (21). Tabulated are the average reduction per iteration of both maximum residual and maximum error, taken over a reduction in residual of at least eleven orders of magnitude. Results from two iterative schemes are shown in Table II: the first using a fixed ω, the near-optimum of which was chosen by trial-and-error; the second in which the ω was chosen by the requirement that the L_2-norm of the residual be minimized at each iteration, given an update "direction." This latter minimum-residual scheme has been found to generally coincide well with the results of Table I, with fixed-ω values near 1 when all eigenvalues lie inside the aformentioned circle; reduced values of ω are required or non-convergence

is seen when conditioning of the operator is inadequate. This test also indicates that $N_{FD} > 1.5\ N_{sp}$ yields good results, with convergence rate increasing as N_{FD} is increased beyond this point. Note also that the convergence rates for the $N_{sp} = 45$, $N_{FD} = 75$ case, where $N_{FD} = 5/3\ N_{sp}$, is faster than the case $N_{sp} = 33$, $N_{FD} = 55$, where the ratio is again 5/3. This indicates that the convergence rate of the scheme does not decrease with spectral mesh refinement, which is a very desirable property.

A similar eigenvalue spectrum behavior is seen in Table III for an application of the present preconditioner to the advection-diffusion problem

$$\begin{aligned} U_x - \nu\, U_{xx} &= f \quad , \quad x \in [-1,1] \\ U(-1) &= a \quad , \quad U(1) = b \end{aligned} \tag{22}$$

The preconditioning operator used here is

$$M = \delta_x^{\circ} - (\varepsilon + \nu)\, \delta_{xx}^{\circ} \tag{23}$$

The results presented in the first part of Table III are for $\varepsilon = \nu = 10^{-3}$; that is, some additional, artificial viscosity is applied in the preconditioning operator augmenting the "natural" viscosity. Again, the preconditioner effectively clusters the eigenvalues of the operator. In the latter part of Table III are shown the eigenvalues for a case with the same physical viscosity, $\nu = 10^{-3}$, but with double the artificial viscosity of the former case; $\varepsilon = 2 \times 10^{-3}$. Only a small effect is seen once $N_{FD} > 1.5\ N_{sp}$.

Maximum and minimum eigenvalues for first-, second- and third-order upwind schemes preconditioning Eq. (22) with $\nu = 10^{-3}$ are shown in Table IV. Note that although the eigenvalues are well-behaved in magnitude, the imaginary parts are considerably larger than in the previous scheme. Convergence-acceleration methods such as minimumal-residual choice of ω are less effective

on operators with eigenvalues having large imaginary parts, or may be unstable or oscillatory during convergence. Such a feature is therefore not good in a preconditioning application.

To illustrate the differences in eigenvalue spectra between the present, central-differences preconditioning and the high-order upwind preconditioning, eigenvalues are plotted in Figs. 1-6 for Eq. 22, $\nu = 10^{-5}$. The artificial viscosity for the central-difference scheme was $\varepsilon = 10^{-4}$. Shown for $N_{sp} = 45$ and 60 are the spectra (positive imaginary parts) without preconditioning (Figs. 1 and 4), for $N_{FD} = 1.2\ N_{sp}$ (Fig. 2a, 5a), for $N_{FD} = 1.5\ N_{sp}$ (Figs. 2b, 5b), and for first-, second-, and third-order upwind (Figs. 3, 6). Note that except for a few spurious eigenvalues, the spectra for central-difference preconditioning are well-contained and have small imaginary parts, whereas those of the upwind preconditioners have large imaginary parts, increasing as the real parts tend to zero. As stated before, such behavior is detrimental to iterative convergence of the overall preconditioned scheme.

Conclusions

We have demonstrated an effective preconditioner for the operators which result from collocation discretization of first derivatives. Results of an eigenvalue analysis of the preconditioned-iteration scheme agreed well with actual results for a simple test problem. The preconditioner performed well, producing rapid convergence.

The proposed scheme is advantageous in application as a preconditioner for spectral collocation solutions of the compressible Navier-Stokes equations for several reasons. First, since solution of the preconditioning equation is related to solution of the original equation via central finite differences,

one can draw on well-developed techniques, such as the Beam and Warming scheme. Second, physically-relevant boundary conditions can be imposed in the preconditioner, so that boundary points are included consistently in the update scheme. For outflow boundaries, when the spectral scheme merely enforces the given equation, it was found that simple extrapolation conditions in the preconditioner were adequate. Finally, it was found that the convergence rate of the preconditioned scheme was essentially independent of the spectral mesh size, when the preconditioning mesh was fine enough. The degree of refinement over the spectral mesh required in the preconditioner was not large; 50% more points was found to be adequate. Such a mesh would still be considered quite coarse from finite-difference standards, so the greatly increased accuracy of the spectral method can be economically realized. It was also shown that high-order upwind schemes, despite a potential advantage in more efficient inversion of the preconditioning operation, probably will not perform as well, due to the large imaginary parts of the eigenvalues of the preconditioned operator.

Currently underway are investigations using two-dimensional test problems to verify in that setting the findings presented here. One aerodynamic application planned is that of supersonic flow over a blunt leading edge, using bow-shock fitting in a compressible Navier-Stokes formulation. This problem has been investigated using a finite-difference Beam and Warming code (ref. 4); it was found that truncation error compromised heat-transfer predictions, thus a higher-order accuracy technique is required. Other applications planned involve basic studies of fluid-dynamic stability of compressible flow, such as high-speed boundary layers. Spectral methods are needed for such simulations due to the stringent accuracy requirements of wave propagation prediction in this setting.

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Table I. Maximum and minimum eigenvalues for various N_{sp}, N_{FD} preconditioning Eq. (21). (Real, imaginary parts)

N_{sp}	N_{FD}	λ_{max}	λ_{min}
9	9	21.48, 0.0	1., 0.0
	14	1.208, ± .046	.339, ± .310
	18	1.054, ± .018	.173, 0.0
17	17	47.46, 0.0	1., 0.0
	21	2.333,0.0	.276, 0.0
	26	1.364,0.0	.218, 0.0
	34	1.136,0.0	.082, 0.0
33	33	56.28, 0.0	1., 0.0
	40	2.178, 0.0	.097, 0.0
	55	1.365, ± .013	.056, 0.0
	66	1.348, 0.0	.049, 0.0
45	45	37.42, 0.0	1., ± .002
	54	2.456, 0.0	.073, 0.0
	68	1.53, 0.0	.049, 0.0
	75	1.390, ± .062	.045, 0.0

Table II. Convergence rates of preconditioned schemes for Eq. (21). $\rho(R)$, $\rho(\Delta)$ are average reduction per iteration of residual, error, respectively.

			fixed ω		MR	
N_{sp}	N_{FD}	ω	$\rho(R)$	$\rho(\Delta)$	$\rho(R)$	$\rho(\Delta)$
9	9		$\geq N \geq$		$\geq N \geq$	$\geq N \geq$
	14	.4	.616	.601	.508	.501
	18	.5	.496	.501	.391	.383
17	17		$\geq N \geq$		$\geq N \geq$	$\geq N \geq$
	21	.5	.828	.834	$\geq N \geq$	$\geq N \geq$
	26	.4	.639	.615	.225	.257
	34	.5	.515	.508		
33	33		$\geq N \geq$		$\geq N \geq$	$\geq N \geq$
	40	.5	.609	.576	$\geq N \geq$	$\geq N \geq$
	55	1.0	.419	.386	$\geq N \geq$	$\geq N \geq$
	66	.9	.267	.207	.120	.112
45	45		$\geq N \geq$		$\geq N \geq$	$\geq N \geq$
	54	.5	.619	.566	$\geq N \geq$	$\geq N \geq$
	68	1.0	.553	.494	.283	.224
	75	1.0	.337	.291	.239	.196

Table III. Maximum and minimum eigenvalues for various N_{sp}, N_{FD} preconditioning Eq. (22). (Real, imaginary parts)

$\nu = 10^{-3}, \quad \varepsilon = 10^{-3}$

N_{sp}	N_{FD}	λ_{max}	λ_{min}
33	33	29.152, 0.0	.753, 0.0
	40	2.190, ± 1.269	.103, 0.0
	55	1.360, ± .056	.067, 0.0
	66	1.169, ± .120	.062, 0.0
45	45	13.024, ± 7.955	.754, 0.0
	54	2.329, ± .140	.124, 0.0
	68	1.502, ± .092	.086, 0.0
	75	1.381, ± .079	.077, 0.0

$\nu = 10^{-3}, \quad \varepsilon = 2 \times 10^{-3}$

N_{sp}	N_{FD}	λ_{max}	λ_{min}
33	33	21.590, 0.0	.997, 0.0
	40	2.316, 0.0	.072, 0.0
	55	1.400, 0.0	.070, 0.0
	66	1.261, 0.0	.046, 0.0
45	45	109.8, 0.0	.998, 0.0
	54	2.709, ± 1.058	.119, 0.0
	68	1.591, 0.0	.082, 0.0
	75	1.454, 0.0	.074, 0.0

Table IV. Maximum and minimum eigenvalues for upwind schemes preconditioning Eq. (22) (Real, imaginary parts)

$\nu = 10^{-3}$

N	order	λ_{max}	λ_{min}
33	1	.231, ± 1.359	.251, ± .448
	2	1.000, 0.0	.173, ± .258
	3	.651, ± 1.843	.685, ±0.0
45	1	.211, ± 1.386	.416, ± .435
	2	1.000, 0.0	.268, ± .285
	3	.547, ± 1.927	.999, 0.0

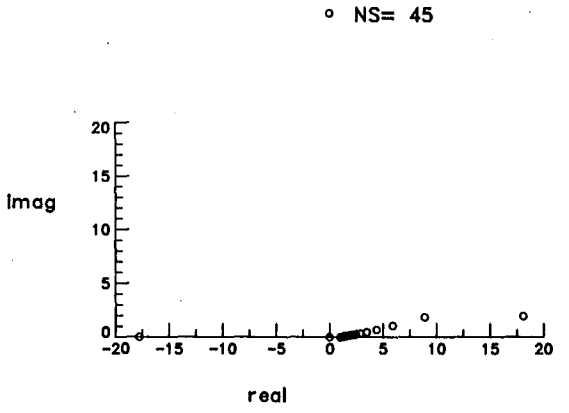
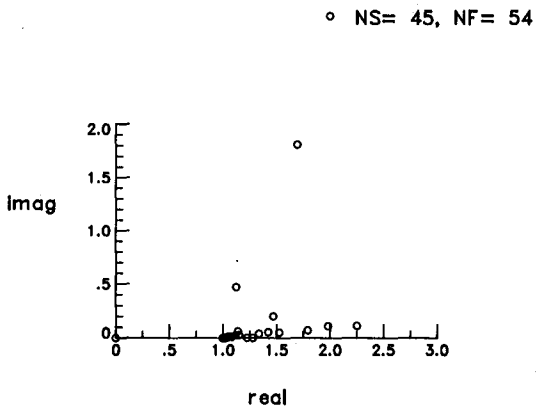
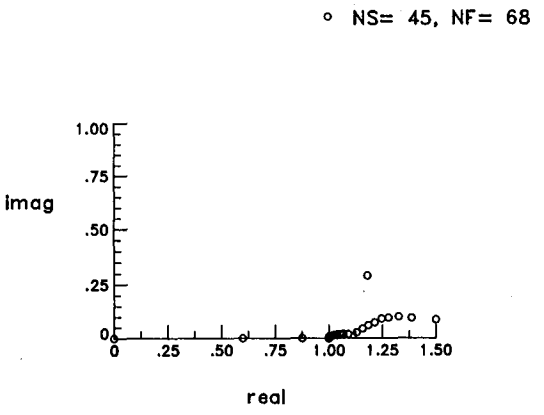


Fig. 1. Eigenvalues of spectral discretization of Eq. 22, $\nu = 10^{-5}$.

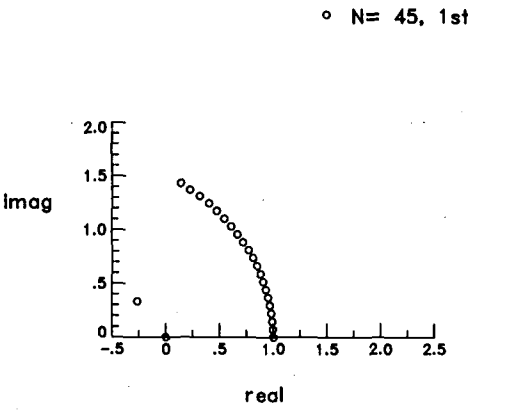


(a) $N_{FD} = 1.2\ N_{SP}$

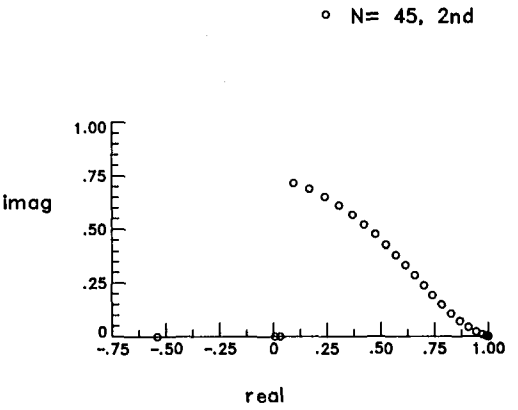


(b) $N_{FD} = 1.5\ N_{SP}$

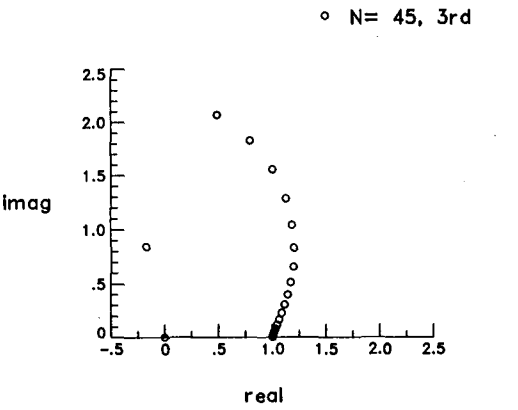
Fig. 2. Eigenvalues of spectral discretization of Eq. 22, $\nu = 10^{-5}$, preconditioned with present scheme, $\varepsilon = 10^{-4}$.



(a) first order



(b) second order



(c) third order

Fig. 3. Eigenvalues of spectral discretization of Eq. 22, $\nu = 10^{-5}$, preconditioned with upwind scheme.

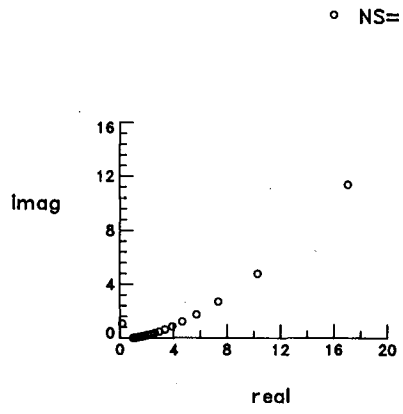
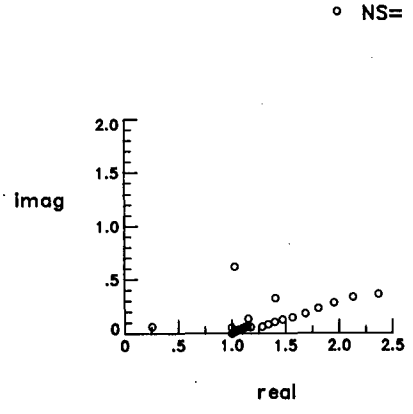
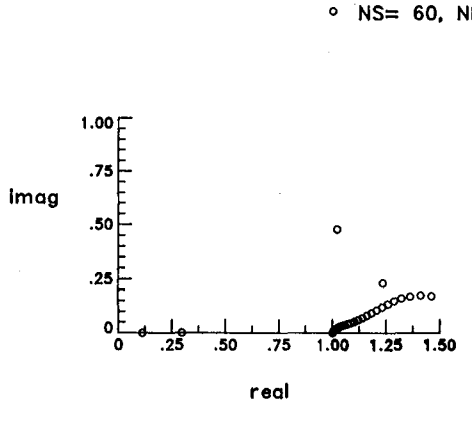


Fig. 4. Eigenvalues of spectral discretization of Eq. 22, $\nu = 10^{-5}$.

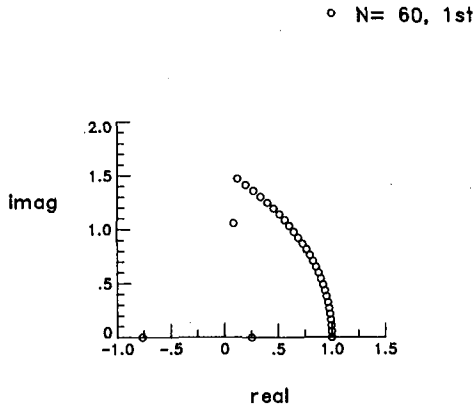


(a) $N_{FD} = 1.2\ N_{SP}$

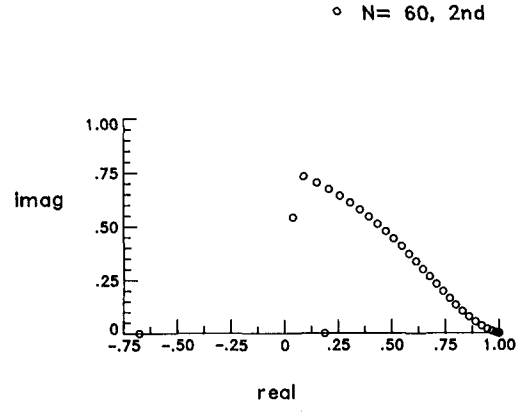


(b) $N_{FD} = 1.5\ N_{SP}$

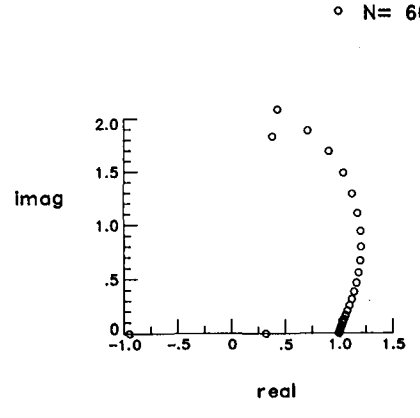
Fig. 5. Eigenvalues of spectral discretization of Eq. 22, $\nu = 10^{-5}$, preconditioned with present scheme, $\varepsilon = 10^{-4}$.



(a) first order



(b) second order



(c) third order

Fig. 6. Eigenvalues of spectral discretization of Eq. 22, $\nu = 10^{-5}$, preconditioned with upwind scheme.

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16. Abstract

Efficient solution of the equations from spectral discretizations is essential if the high-order accuracy of these methods is to be realized. Direct solution of these equations is rarely feasible, thus iterative techniques are required. A preconditioning scheme for first-order Chebyshev collocation operators is proposed herein, in which the central finite difference mesh is finer than the collocation mesh. Details of the proper techniques for transferring information between the meshes is given here, and the scheme is analyzed by examination of the eigenvalue spectra of the preconditioned operators. The effect of artificial viscosity required in the inversion of the finite difference operator is examined. A second preconditioning scheme, involving a high-order upwind finite difference operator of the van Leer type is also analyzed to provide a comparison with the present scheme. Finally, the performance of the present scheme is verified by application to several test problems.

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